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Dynamic analysis of functionally graded (FG) cylindrical shells by nonlinear grading pattern using meshless local Petrov-–Galerkin (MLPG) method[1]

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Abstract. In this research, the dynamic stress equilibrium equations are derived in the polar coordinates for functionally graded (FG) cylindrical shell based on meshless local Petrov-–Galerkin (MLPG) method. The present method is a true meshless method. By means of the radial basis functions, all the requirements for an effective and suitable shape function are established. Hence in this study, the multiquadrics (MQ) radial basis functions are exploited as the shape function governing the problem. The equations derived based on meshless local Petrov-–Galerkin (MLPG) method have the capabilities to analyze the cylindrical shell with the homogeneous, heterogeneous and also FG materials. To simulate the mechanical properties of functionally graded material (FGM), the nonlinear volume fractions model is used on radial direction. In order to calculate the available integrals, the Gaussian quadrature is applied. To demonstrate the capability of the relations derived based on meshless local Petrov-–Galerkin (MLPG) method for dynamic analysis of FG cylindrical shell, a cylinder is analyzed with different volume fraction exponents under the harmonic and rectangular shock loading. The present method furnishes a ground for a more flexible design.

Key words. Meshless method, Petrov—Galerkin method, cylindrical shell, dynamic analysis, radial basis function, functionally graded materials (FGM), shock loading.

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1. Introduction

Some numerical analysis of cylindrical shell can found in [1—3] in particular for the free vibration and transient analysis [4], transient analysis of thermo–elastic waves [5], vibration characteristics of FGM cylindrical shells and elastic mechanical stress analysis. In recent decade, meshless local Petrov—Galerkin (MLPG) method has become very useful and effective solving method in cylindrical shell for functionally graded material because these materials have variable mechanical properties and this method does not require to the mesh generation on the domain.

In this research, at first the equation governing the dynamic behavior of cylindrical shells made of functionally graded material derive in the polar coordinates using meshless local Petrov—Galerkin (MLPG) method. Then, to verify the present method for dynamic analyses cylindrical shells, the obtain results in this study are compared with the analytical method and the finite element method (FEM). At the end, the cylindrical shell under the harmonic and rectangular shock loading will be analyzed for the various values of volume fraction exponent and the results gained will be mentioned.

2. The meshless implementation of cylindrical shell

Some kinds of long cylindrical shells like tubes are considered with axial symmetry in terms of boundary conditions and loading. The equilibrium equation for these structures in polar coordinates can be written as follows:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\left(\sigma_r - \sigma_\theta\right) = \rho \frac{\partial^2 u_r}{\partial t^2}\,, \tag{1}$$

where σ_r and σ_θ are radial and hoop stress respectively. The term u_r denote the radial displacement.

Due to lack of access to the exact amount of the displacement function, it will be necessary to approximate this function at the nodes using the shape functions. The following equation is applied to approximate the field function:

$$\hat{u} = u \cong \sum_{i=1}^{n} \Phi_i\left(r\right).\hat{u}_i\left(t\right) = \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_{n-1} & \varphi_n \end{bmatrix} \begin{Bmatrix} u_1\left(t\right) \\ u_2\left(t\right) \\ \vdots \\ u_{n-1}\left(t\right) \\ u_n\left(t\right) \end{Bmatrix} = \boldsymbol{\Phi} \times \boldsymbol{U}(t)\,, \tag{2}$$

where $\boldsymbol{\Phi}$ is the matrix of shape function and $\boldsymbol{U}(t)$ is the displacement vector for all the nodes at time t. In order to demonstrate the high ability of created relation, analysis of cylindrical shells made with functionally graded materials is added to this relationship. Volume fraction model is used to define the mechanical properties of functionally graded material. The strains and the stresses in terms of displacement of the nodes are obtained as follows:

$$\begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \end{bmatrix} \times \hat{u} = \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \end{bmatrix} \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_{n-1} & \varphi_n \end{bmatrix} \begin{Bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_{n-1}(t) \\ u_n(t) \end{Bmatrix} =$$

$$= \boldsymbol{BU}(t)\,, \tag{3}$$

$$\sigma = \boldsymbol{D} \times \begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \end{Bmatrix} = \boldsymbol{DBU}(t)\ , \tag{4}$$

where the matrix of materials D and matrix B are as follows:

$$\boldsymbol{D} = \left(\frac{E}{(1+\nu)(1-2\nu)} \right) \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix}, \tag{5}$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{\partial \varphi_1}{\partial r} & \frac{\partial \varphi_2}{\partial r} & \dots & \frac{\partial \varphi_{n-1}}{\partial r} & \frac{\partial \varphi_n}{\partial r} \\ \frac{\varphi_1}{\partial r} & \frac{\varphi_2}{\partial r} & \dots & \frac{\varphi_{n-1}}{\partial r} & \frac{\varphi_n}{\partial r} \end{bmatrix}. \tag{6}$$

3. Verification

In numerical methods, the certainty of accuracy of the results obtained in each method has utmost importance. In the following, the present method will be compared with the analytical method.

3.1. Verification with the analytical method

In this section, the results of local Petrov–Galerkin method are compared to the analytical solution. For this purpose, a hollow cylinder is considered with inner radius r_{in} and outer radius r_{out} under internal pressure P_{in} and external pressure P_{out} (see Fig. 1).

In this study, the multiquadrics (MQ) radial basis shape function is used for the establishment of relations govern the problem. To calculate the available integrals, the Gaussian quadrature is used.

According to the analytical solution [2], the radial stress, hoop stress and displacement are as follows:

$$\sigma_{rr} = \frac{r_{\text{in}}^2 r_{\text{out}}^2 \left(P_{\text{out}} - P_{\text{in}}\right)}{r_{\text{out}}^2 - r_{\text{in}}^2} \frac{1}{r^2} + \frac{r_{\text{in}}^2 P_{\text{in}} - r_{\text{out}}^2 P_{\text{out}}}{r_{\text{out}}^2 - r_{\text{in}}^2}\,, \tag{7}$$

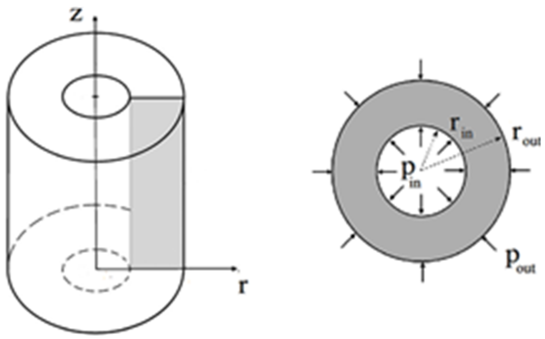

Fig. 1. Geometry of the hollow cylinder

$$\sigma_{\theta\theta} = -\frac{r_{\text{in}}^2 r_{\text{out}}^2 \left(P_{\text{out}} - P_{\text{in}}\right)}{r_{\text{out}}^2 - r_{\text{in}}^2}\frac{1}{r^2} + \frac{r_{\text{in}}^2 P_{\text{in}} - r_{\text{out}}^2 P_{\text{out}}}{r_{\text{out}}^2 - r_{\text{in}}^2}, \tag{8}$$

$$u_{rr} = \frac{1+\nu}{E}\left[\frac{r_{\text{in}}^2 r_{\text{out}}^2 \left(P_{\text{out}} - P_{\text{in}}\right)}{r_{\text{out}}^2 - r_{\text{in}}^2}\frac{1}{r} + (1-2\nu)\frac{r_{\text{in}}^2 P_{\text{in}} - r_{\text{out}}^2 P_{\text{out}}}{r_{\text{out}}^2 - r_{\text{in}}^2} r\right]. \tag{9}$$

For verification, a cylinder with the inner radius $r_{\text{in}} = 0.25\,\text{m}$ and the outer radius $r_{\text{out}} = 0.5\,\text{m}$ made of aluminum with elastic modulus $E = 70\,\text{GPa}$ and Poisson's ratio $\nu = 0.3$ is considered.

The loading function (internal pressure) is expressed as

$$P_{in} = P_0\left(1 - \mathrm{e}^{-c_0 t}\right). \tag{10}$$

In this equation, $P_0 = 20\,\text{MPa}$ and $c_0 = 10^2\,\text{s}^{-1}$ are assumed.

The results of the meshless method are compared to the analytical solution in Figs. 2 and 3. As can be seen in these figures, the results of the meshless method show a good agreement to the results of the analytical solution. In Table 1, the percentage errors for local Petrov–Galerkin method and analytical solution in middle point of thickness of the cylinder ($r = 0.375\,\text{m}$) are shown. Table 1. indicates that the relation done with local Petrov–Galerkin method has a very high accuracy, thus this method can be used as a practical approach for dynamic analysis of cylindrical shells.

Table 1. Comparison of obtained results from meshless method (local Petrov–Galerkin method) with those obtained by analytical method for middle point of thickness of cylinder

Variable	Analytical method	Local Petrov-Galerkin method	Percentage error
Displacement (m)	$1.0111\cdot10^{-4}$	$1.0112\cdot10^{-4}$	$9.89\cdot10^{-3}$
Radial stress (Pa)	$-5.185\cdot10^{6}$	$-5.139\cdot10^{6}$	0.8872
Hoop stress (Pa)	$1.852\cdot10^{7}$	$1.842\cdot10^{7}$	0.5399

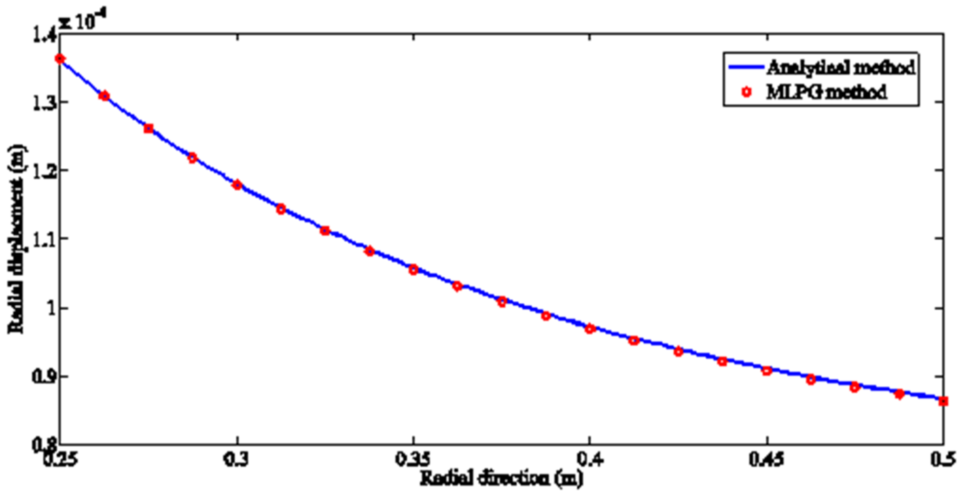

Fig. 2. Comparison of displacement obtained through the meshless method with the analytical solution

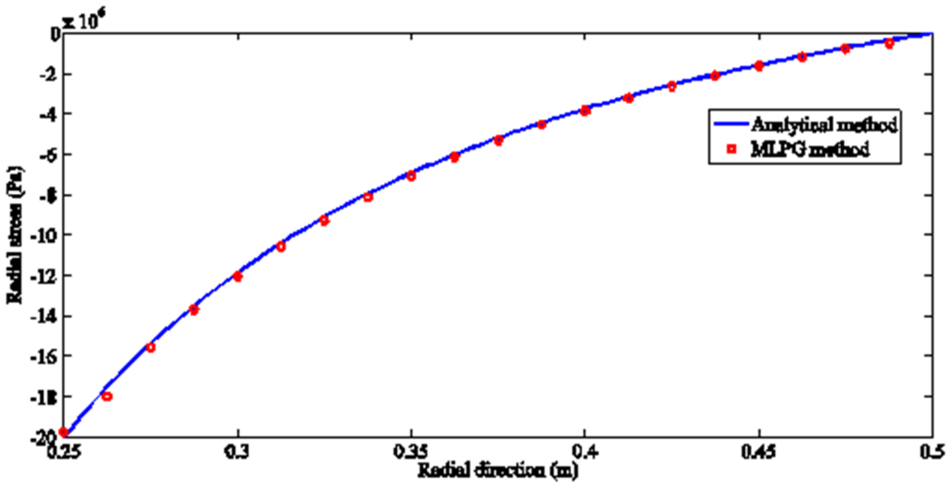

Fig. 3. Comparison of radial stress obtained through the meshless method with the analytical solution

4. Numerical example

In this section, we analyze the cylindrical shell made of functionally graded material under two different types of shock loading (harmonic and rectangular) with the meshless method.

4.1. Harmonic shock loading

A cylindrical shell is considered with inner radius $r_{\mathrm{in}} = 0.5\,\mathrm{m}$ and outer radius $r_{\mathrm{out}} = 1.0\,\mathrm{m}$ under harmonic internal pressure as follows:

$$P(t) = \begin{cases} P_0 \sin\left(10^4 \pi t\right) & t \leq 0.005 \\ 0 & t > 0.005 \end{cases}, \tag{11}$$

where $P_0 = 5\,\mathrm{MPa/s}$ is assumed.

The material properties of this cylinder are displayed in Table 2. Displacement, radial stress and hoop stress for the various values of volume fraction exponent in the middle point of thickness of the cylinder $r = 0.75\,\mathrm{m}$ are drawn in Figs. 4–6.

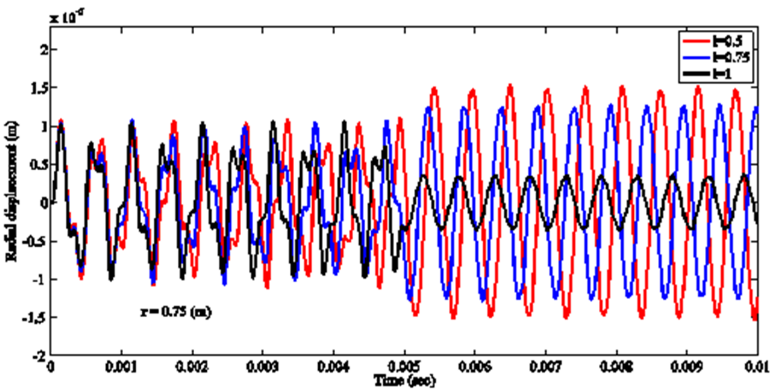

Fig. 4. Amounts of displacement in the middle point of thickness of the cylinder for the various values of volume fraction exponent under harmonic shock loading

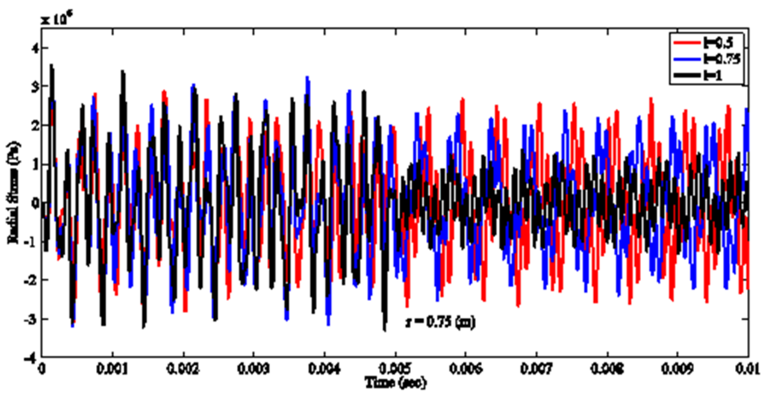

Fig. 5. Amounts of radial stress in the middle point of thickness of the cylinder for the various values of volume fraction exponent under harmonic shock loading

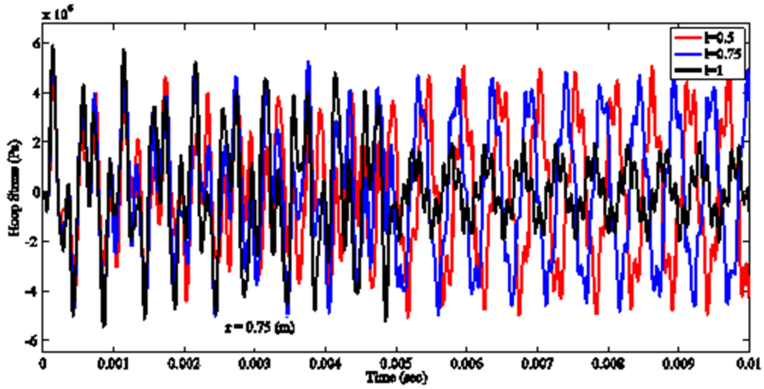

Fig. 6. Amounts of hoop stress in the middle point of thickness of the cylinder for the various values of volume fraction exponent under harmonic shock loading

4.2. Rectangular shock loading

A cylindrical shell is considered with inner radius $r_{\text{in}} = 0.5\,\text{m}$ and outer radius $r_{\text{out}} = 1.0\,\text{m}$ under internal pressure $P(t)$ as follows:

$$P(t) = \begin{cases} P_0 & t \leq 0.005 \text{ sec} \\ 0 & t > 0.005 \text{ sec} \end{cases}, \tag{12}$$

where $P_0 = 5\,\text{MPa/s}$ is presumed.

The mechanical features of this cylinder are shown in Table 2. In Fig. 7, the amounts of displacement for the various values of volume fraction exponent in the middle point of thickness of the cylinder $r = 0.75\,\text{m}$ are exhibited. By augmenting the value of volume fraction exponent, the maximum displacement decreases and the frequency raises.

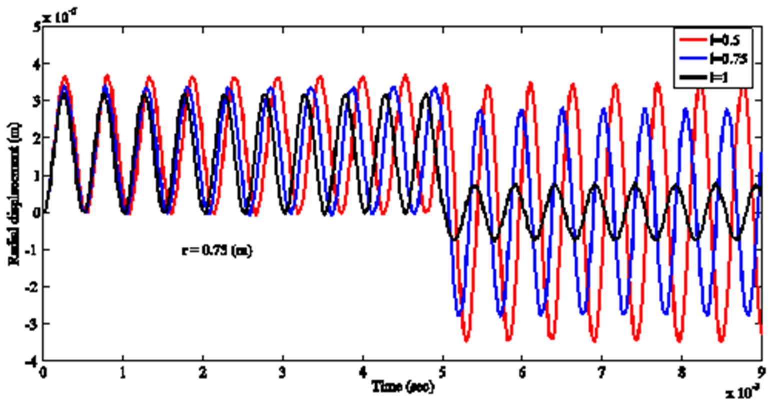

Fig. 7. Amounts of displacement in the middle point of thickness of the cylinder for the various values of volume fraction exponent under rectangular shock loading

5. Conclusion

In this paper, for dynamic analysis of cylindrical shell made of functionally graded materials (FGM), meshless local Petrov–Galerkin (MLPG) method has been exploited. To simulate the mechanical properties of functionally graded materials, volume fraction model has been applied. Capability and high accuracy of the presented method was determined by comparing the results of this method to the analytical method and the finite element method. Some of the significant results of this research are summarized as follows:

- The local Petrov–Galerkin method has the capability to analyze of cylindrical shells made of functionally graded material with high accuracy.
- The obtained results with present method in comparison to the analytical method and the finite element method (FEM) show good agreement, furthermore this comparison can demonstrate capability and high accuracy of the present method.
- The displacement, radial stress and hoop stress under two types of shock loading (harmonic and rectangular) and for the various values of volume fraction exponent showed, the maximum amount of displacement by increasing the value of volume fraction exponent decreases and the frequency along with this trend raises. Likewise it was determined that the maximum radial and hoop stresses by rising the value of volume fraction exponent when shock loading enters increase and then in the phase of free vibration are decreased.

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